BCS-BEC crossover in a relativistic boson-fermion model beyond mean field approximation

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We investigate the fluctuation effect of the di-fermion field in the crossover from Bardeen-Cooper-Schrieffer (BCS) pairing to a Bose-Einstein condensate (BEC) in a relativistic superfluid. We work within the boson-fermion model obeying a global U(1) symmetry. To go beyond the mean field approximation we use Cornwall-Jackiw-Tomboulis (CJT) formalism to include higher order contributions. The quantum fluctuations of the pairing condensate is provided by bosons in non-zero modes, whose interaction with fermions gives the two-particle-irreducible (2PI) effective potential. It changes the crossover property in the BEC regime. With the fluctuations the superfluid phase transition becomes the first order in grand canonical ensemble. We calculate the condensate, the critical temperature T_c and particle abundances as functions of crossover parameter the boson mass.

I. INTRODUCTION

Fermion pairings are mechanisms for superconductivity and superfluidity. In the case of weak attractive interaction between fermions the pairings are well described in Bardeen-Cooper-Schrieffer (BCS) theory [1], where fermion pairs are typically of a size much larger than the mean interparticle distance. In some sense the pairing can be regarded as taking place in momentum space. As the attractive interaction gets strong enough fermion pairs become real bosonic bound states. Below a critical temperature macroscopically large number of these molecular bosons occupy the ground state and form a Bose-Einstein condensate (BEC). The idea of covering BEC regime in an extended BCS theory was proposed by Eagles [2] and Leggett [3]. A quantitative description of the crossover at finite temperatures from weak to strong coupling regime was given by Nozieres and Schmitt-Rink [4], who wrote down the universal pair wave function which can be reduced to the correct ground states at BCS and BEC limit. The BCS-BEC crossover have been extensively investigated in cold atom systems since such a crossover can be observed in experiments by tuning the magnetic field around Feschbach resonance to achieve situations with different scattering length among fermionic atoms [5, 6, 7, 8, 9].

Recently there is a growing interest in extending the theory of the BCS-BEC crossover to relativistic systems. The crossover from pion condensation to Cooper pairing of quarks and antiquarks at large isospin densities provides such an example [10, 11, 12, 13]. Another example is color superconductivity where quarks form Cooper pairs on the Fermi surface due to an attractive interaction mediated by gluon exchange [14, 15, 16, 17, 18, 19, 20, 21, 23] (for reviews, see, for instance, [24, 25, 26, 27, 28, 29, 30, 31]). Because of asymptotic freedom, color superconductivity at very large densities can be studied in a weak-coupling approach [20, 21, 22, 23, 32, 33, 34, 35] using perturbative QCD (see, for example, [36, 37, 38]). For moderate densities more phenomenological models such as the Nambu-Jona-Lasinio (NJL) model [29, 39] or self-consistent Dyson-Schwinger equations [40, 41] are needed. All these approaches usually assume BCS-like picture. When the coupling strength turns even stronger, the BEC-like diquark pairing may set in. A lot of authors used the NJL model to study the BCS-BEC crossover in relativistic superconductors or superfluids [42, 43, 44, 45, 46, 47, 48]. In our previous work we set up a theory for the crossover in relativistic superfluids which includes both bosonic and fermionic degrees of freedom [49]. It is inspired by the boson-fermion model of superconductivity [50, 51, 52]. We only addressed the mean-field approximation of the model. The crossover is realized by tuning the difference between the boson mass and boson chemical potential.

In this paper we extend our previous work [49] by systematic incorporating the fluctuation effect in the crossover from propagating modes of the di-fermion field within Cornwall-Jackiw-Tomboulis formalism (CJT) [53]. Our starting point is the boson-fermion model with a global U(1) symmetry. Similar efforts have been made in the NJL model [45, 48]. The CJT formalism has been employed to analyze fluctuation effects in color superconductors [54, 55]. The inclusion of fluctuation contributions leads to the change of the second order phase transition of normal to superfluid to a first order one. The paper is organized as follows. In Sec. II we re-write the boson-fermion model in terms of the Higgs and Nambu-Goldstone fields. In Sec. III the effective potential is evaluated up to two-loops including the mean field as well as the fluctuation contributions. The Dyson-Schwinger (DS) equations are derived from the effective potential in Sec. IV. The gap and charge density equations are obtained in Sec. V by taking derivatives of the effective potential with respect to the diquark condensate and the chemical potential respectively. The numerical results are presented and analyzed in Sec. VI. Finally we discuss about our results and draw some conclusions in the last section.

Our convention for the metric tensor is $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. Our units are $\hbar = c = k_B = 1$. Four-

vectors are denoted by capital letters, $K \equiv K^\mu = (k_0, \mathbf{k})$ with $k = |\mathbf{k}|$. Fermionic Matsubara frequencies are $\omega_n = ik_0 = (2n+1)\pi T$, while bosonic ones are $\omega_n = ik_0 = 2n\pi T$ with the temperature T and n an integer. Our convention for the Feynman rules in finite temperature and density field theory are chosen as compared to those at zero temperature and density, where the propagator and the vertex for a quantum field $\phi(X)$ (X is a real time-space coordinate) in the theory are like, e. g. $iG(X_1, X_2) = \langle T\phi(X_1)\phi(X_2)\rangle$ and $i\Gamma = i\delta^n/[\delta\phi(X_1)\delta\phi(X_2)\cdots\delta\phi(X_n)]$. At finite temperature and density, we replace $iG \to -G$ and $i\Gamma \to \Gamma$. The energy unit is chosen to be the fermion momentum cutoff Λ appearing in fermion momentum integrals.

II. BOSON-FERMION MODEL

In order to describe the fluctuation effects from the propagating modes of di-fermion bosons, we start from the following Lagrangian which respects the global U(1) symmetry corresponding to total particle number conservation,

$$\begin{aligned}\mathcal{L}(\Phi,\Psi) &= -\frac{1}{2}\overline{\Psi}S_0^{-1}\Psi + |(\partial_\nu - i\mu_b\delta_{\nu 0})\Phi|^2 - m_b^2|\Phi|^2 \\ &\quad +\frac{1}{2}\left(\Phi^\dagger\overline{\Psi}\widehat{\Gamma}\Psi - \Phi\overline{\Psi}\widehat{\Gamma}^\dagger\Psi\right). \end{aligned} \tag{1}$$

In the first term or the fermion sector, $\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}$ and $\overline{\Psi} = (\overline{\psi}, \overline{\psi}_C)$ are spinors for fermions in the Nambu-Gorkov (NG) basis, where the charge conjugate spinor is defined by $\psi_C = C\overline{\psi}^T$ and $\overline{\psi}_C = \psi^T C$ with $C = i\gamma^2\gamma^0$. In momentum space the inverse fermion propagator S_0^{-1} is given by

$$S_0^{-1}(P) = -\begin{pmatrix} \gamma_\mu P^\mu + \mu\gamma^0 - m & 0 \\ 0 & \gamma_\mu P^\mu - \mu\gamma^0 - m \end{pmatrix}, \tag{2}$$

where μ and m are the chemical potential and the mass of fermions respectively. In the second and third terms of the Lagrangian (1) the boson chemical potential and the mass are denoted by $\mu_b = 2\mu$ and m_b. In the boson-fermion interaction part, the last term of the Lagrangian (1), we have defined the interaction vertices

$$\widehat{\Gamma} = 2gi\gamma_5 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \widehat{\Gamma}^\dagger = -2gi\gamma_5 \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{3}$$

with g the Yukawa coupling constant. We can write

$$\Phi = \varphi + \varphi_0 \equiv \frac{1}{\sqrt{2}}(\varphi_R + i\varphi_I) + \varphi_0,$$

where φ_0 is the expectation value of the vacuum or the zero mode of the boson field and $\varphi_{R,I}$ denote the real and imaginary parts of non-zero modes. With φ_0 the U(1) symmetry is spontaneously broken. To make our approach transparent we did not include in Eq. (1) the self-interaction of Φ such as Φ^4, which does not change the results qualitatively. The quartic or higher terms in Δ (or φ_0) are present automatically from the effective potential. The Lagrangian (1) then becomes

$$\begin{aligned}\mathcal{L}(\Delta,\varphi,\Psi) &= -\frac{1}{2}\overline{\Psi}S^{-1}\Psi + \frac{\mu_b^2 - m_b^2}{4g^2}|\Delta|^2 + |(\partial_t - i\mu_b)\varphi|^2 - |\nabla\varphi|^2 - m_b^2|\varphi|^2 + \frac{1}{2}\left(\varphi^\dagger\overline{\Psi}\widehat{\Gamma}\Psi - \varphi\overline{\Psi}\widehat{\Gamma}^\dagger\Psi\right) \\ &= -\frac{1}{2}\overline{\Psi}S^{-1}\Psi + \frac{\mu_b^2 - m_b^2}{4g^2}|\Delta|^2 - \frac{1}{2}(\varphi_R, \varphi_I)D^{-1}\begin{pmatrix} \varphi_R \\ \varphi_I \end{pmatrix} + \frac{1}{2}(\varphi_R, \varphi_I)\overline{\Psi}\begin{pmatrix} \widehat{\Gamma}_R \\ \widehat{\Gamma}_I \end{pmatrix}\Psi. \end{aligned} \tag{4}$$

The action is $I(\Delta,\varphi,\Psi) = \int_X \mathcal{L}(\Delta,\varphi,\Psi)$. The boson-fermion vertices in the second equality are defined by

$$\begin{aligned}\hat{\Gamma}_R &= \sqrt{2}(\widehat{\Gamma} - \widehat{\Gamma}^\dagger) = i2\sqrt{2}g\gamma_5\sigma_1^{NG}, \\ \hat{\Gamma}_I &= -i\sqrt{2}(\widehat{\Gamma} + \widehat{\Gamma}^\dagger) = -i2\sqrt{2}g\gamma_5\sigma_2^{NG},\end{aligned}$$

where $\sigma_{1,2}^{NG}$ are Pauli matrices in the Nambu-Gorkov basis. The inverse fermion and boson propagators S^{-1} and D^{-1} are given in momentum space by

$$\begin{aligned}S^{-1} &= -\begin{pmatrix} \gamma_\mu P^\mu + \mu\gamma^0 - m & i\gamma_5\Delta \\ i\gamma_5\Delta^* & \gamma_\mu P^\mu - \mu\gamma^0 - m \end{pmatrix}, \\ D^{-1} &= -\begin{pmatrix} P_\mu P^\mu + \mu_b^2 - m_b^2 & 2\mu_b i p_0 \\ -2\mu_b i p_0 & P_\mu P^\mu + \mu_b^2 - m_b^2 \end{pmatrix}, \end{aligned} \tag{5}$$

where $\Delta = 2g\varphi_0$ is the condensate. We have dropped the mixing terms of zero and non-zero boson modes since they vanish when carrying out the path integral. We see in (4) that the di-fermion boson is described by two real scalar field.

The Lagrangian (1) can also be expressed in terms of the Higgs and the Nambu-Goldstone fields by parametrizing the complex boson field as $\Phi = \frac{1}{\sqrt{2}}(\eta + \eta_0)e^{2i\theta}$ with $\eta + \eta_0 = \sqrt{2}|\Phi|$ and $2\theta = \arg(\Phi)$. Here the phase field θ is the Nambu-Goldstone field and η the Higgs one. We can choose the unitary gauge by rewriting the fermion field in the form and $\Psi = \begin{pmatrix} e^{i\theta}\psi \\ e^{-i\theta}\psi_C \end{pmatrix}$. Inserting the above expression for Φ and Ψ into the Lagrangian (1), we obtain

$$\begin{aligned}\mathcal{L}(\Delta, \varphi, \Psi) &= -\frac{1}{2}\overline{\Psi}S^{-1}\Psi + \frac{\mu_b^2 - m_b^2}{4g^2}\Delta^2 + \frac{1}{2}\left[(\partial_\nu\eta)^2 + (\mu_b^2 - m_b^2)\eta^2\right] \\ &+2(\eta + \eta_0)^2\left[(\partial_\nu\theta)^2 - \mu_b\partial_0\theta\right] - \frac{1}{2}(\partial^\mu\theta)\overline{\Psi}\sigma_3\gamma_\mu\Psi + \frac{1}{2}\eta\overline{\Psi}\widetilde{\Gamma}\Psi, \end{aligned} \tag{6}$$

where S^{-1} is given by Eq. (5) but with $\Delta = \sqrt{2}g\eta_0$ which is different from the case of (4). Note that in the Lagrangian (6) we still use the same symbol Ψ and $\overline{\Psi}$ but now without the phase θ: $\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}$ and $\overline{\Psi} = (\overline{\psi}, \overline{\psi}_C)$. In the second to last term σ_3 is the Pauli matrix in the NG basis. In the last term we have defined a new vertex

$$\widetilde{\Gamma} \equiv \sqrt{2}ig\gamma_5 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

We can see that the mass term of θ is absent in the tree level as expected for the Nambu-Goldstone mode.

A few remarks about the Lagrangian (4) and (6) are needed. We see that the complex boson field φ in (4) is transformed into the Higgs and Nambu-Goldstone fields in the Lagrangian (6). Both (4) and (6) are parametrizations of the same Lagrangian (1) which are equivalent to each other. We choose (4) in this paper as it is more simple and transparent than (6). For example, in (4), in addition to the 2PI diagrams from the fermion and Higgs fields, there are also diagrams from the fermion and Nambu-Goldstone fields from (6), which are much more complicated. Another example is that the only two-point Green function for bosons from (4) is $\langle P\varphi(X)\varphi^\dagger(Y)\rangle \sim \langle P\eta(X)e^{2i\theta(X)}\eta(Y)e^{-2i\theta(Y)}\rangle$, where P denotes the path-ordered operator, contains the correlations of both the Higgs and Nambu-Goldstone fields.

Note that we used the roman letter S to denote the bare fermion propagator but with the condensate Δ, which is also called the tree level propagator. We will use calligraphic letter $\mathcal{S}$ to denote the fully dressed fermion propagator.

III. CJT FORMALISM

We start from the Lagrangian (4). The CJT effective potential reads [53],

$$\begin{aligned}\Gamma(\Delta, \overline{\mathcal{D}}, \overline{\mathcal{S}}) &= -I(\Delta) + \frac{1}{2}\Big\{\mathrm{Tr}\ln\overline{\mathcal{D}}^{-1} + \mathrm{Tr}(D^{-1}\overline{\mathcal{D}} - 1) \\ &-\mathrm{Tr}\ln\overline{\mathcal{S}}^{-1} - \mathrm{Tr}(S^{-1}\overline{\mathcal{S}} - 1) - 2\Gamma_{2PI}(\overline{\mathcal{D}}, \overline{\mathcal{S}})\Big\}, \end{aligned} \tag{7}$$

where $\overline{\mathcal{D}}, \overline{\mathcal{S}}$ are full boson and fermion propagators and $I(\Delta) \equiv I(\Delta, 0, 0)$ is the tree-level action with all fields replaced by their expectation values (we have assumed $\langle\varphi\rangle = \langle\Psi\rangle = \langle\overline{\Psi}\rangle = 0$). The factor 1/2 for the fermion term $-\mathrm{Tr}\ln\overline{\mathcal{S}}^{-1}$ comes from the doubling of degrees of freedom in NG basis. D^{-1} and S^{-1} the inverse tree-level propagators for bosons and fermions,

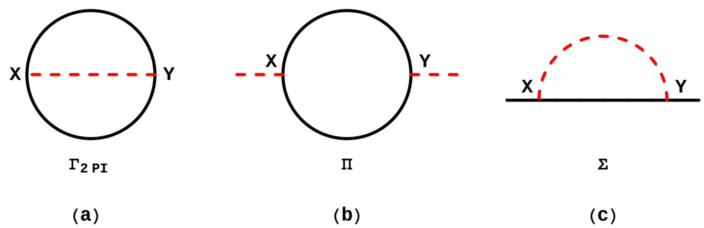
$$D_{ij}^{-1} = -\frac{\delta^2 I}{\delta\varphi_i(X)\delta\varphi_j(Y)}, \tag{8}$$

with $i, j = R, I$ and S^{-1} is given by Eq. (5). Note that all propagators differ their normal forms by a sign, i.e. $S \to -S$ and $D \to -D$ etc. The contribution of 2-particle irreducible vacuum diagrams Γ_2 is truncated to the 2-loop level as shown in Fig. 1,

$$\Gamma_{2PI}(\overline{\mathcal{D}}, \overline{\mathcal{S}}) \approx -\frac{1}{4}\mathrm{Tr}\left\{\overline{\mathcal{D}}_{XY}\mathrm{Tr}\left[\widehat{\Gamma}_X\overline{\mathcal{S}}_{XY}\widehat{\Gamma}_Y\overline{\mathcal{S}}_{YX}\right]\right\}, \tag{9}$$

where X, Y denote indices of color, flavor, space, Nambu-Gorkov and/or bosonic species, and the trace is of functional sense running over the general indices. The full propagators are denoted by calligraphic letters. The negative sign

Figure 1: (a) The 2PI vacuum diagrams up to 2 loops. (b) The boson self-energy derived from the 2PI vacuum diagrams. (c) The fermion self-energy derived from the 2PI vacuum diagrams. The solid and dashed lines denote fermion and boson full propagators. X, Y label indices of the color, flavor, space and/or bosonic species.



comes from the quark loop. A factor of $1/2$ is from the Taylor expansion of the interaction and the other one is from the NG basis.

The Dyson-Schwinger (DS) equations for fermions and bosons can be derived by taking derivatives of the effective potential with respect to propagators,

$$\begin{aligned} \left.\frac{\delta\Gamma}{\delta\overline{\mathcal{D}}}\right|_{\overline{\mathcal{D}}=\mathcal{D},\overline{\mathcal{S}}=\mathcal{S}} &= 0, \\ \left.\frac{\delta\Gamma}{\delta\overline{\mathcal{S}}}\right|_{\overline{\mathcal{D}}=\mathcal{D},\overline{\mathcal{S}}=\mathcal{S}} &= 0, \end{aligned} \tag{10}$$

which lead to the DS equations,

$$\begin{aligned} \mathcal{D}^{-1} &= D^{-1} - 2\frac{\delta\Gamma_{2PI}}{\delta\mathcal{D}} = D^{-1} + \Pi(\mathcal{D},\mathcal{S}), \\ \mathcal{S}^{-1} &= S^{-1} + 2\frac{\delta\Gamma_{2PI}}{\delta\mathcal{S}} = S^{-1} - \Sigma(\mathcal{D},\mathcal{S}). \end{aligned} \tag{11}$$

where the self-energies for bosons and fermions Π and Σ come from Γ_{2PI} in Eq. (9) and are shown in Fig. 1(b) and (c) respectively. They are expressed by,

$$\begin{aligned} \Pi(\mathcal{S},\mathcal{D}) &= \frac{1}{2}\mathrm{Tr}[\widehat{\Gamma}\mathcal{S}\widehat{\Gamma}\mathcal{S}], \\ \Sigma(\mathcal{D},\mathcal{S}) &= \mathrm{Tr}[\mathcal{D}\widehat{\Gamma}\mathcal{S}\widehat{\Gamma}]. \end{aligned} \tag{12}$$

Substituting the DS equations (11) into Eq. (7), we arrive at

$$\begin{aligned} \Gamma(\Delta,\mathcal{D},\mathcal{S}) &= -I(\Delta) + \frac{1}{2}\left\{\mathrm{Tr}\ln\mathcal{D}^{-1} + \mathrm{Tr}[\mathcal{D}^{-1}\mathcal{D} - \Pi(\mathcal{D},\mathcal{S})\mathcal{D} - 1]\right. \\ &\quad \left. -\mathrm{Tr}\ln\mathcal{S}^{-1} - \mathrm{Tr}[\mathcal{S}^{-1}\mathcal{S} + \Sigma(\mathcal{D},\mathcal{S})\mathcal{S} - 1] - 2\Gamma_{2PI}(\mathcal{D},\mathcal{S})\right\} \\ &= -I(\Delta) + \frac{1}{2}\left\{\mathrm{Tr}\ln\left[D^{-1}(1 + D\Pi)\right] - \mathrm{Tr}\ln\left[S^{-1}(1 - S\Sigma)\right] - \mathrm{Tr}(\Sigma\mathcal{S})\right\} \\ &\approx -I(\Delta) + \frac{1}{2}\left\{\mathrm{Tr}\ln D^{-1} - \mathrm{Tr}\ln S^{-1} + \mathrm{Tr}[D\Pi(\mathcal{D},\mathcal{S})]\right\}. \end{aligned} \tag{13}$$

Hereafter we use the thermodynamical potential Ω to replace the effective potential Γ, $\Omega = \Omega_0 - \Gamma_{2PI}$, where the sum of first three terms in Eq. (13) are denoted as Ω_0. Note that the expansion in terms of self-energies made in the last line somehow leads to a partial loss of self-consistency, but it much simplifies the numerical calculation. One can also make an expansion in terms of the condensate Δ near the critical temperature where Δ is small [54, 55].

IV. PSEUDOGAP AND RENORMALIZED BOSON MASS

In this section we address the DS equations (11) for bosons and fermions. First we address the DS equation for fermions. When obtaining $\mathcal{S}$ from elements of $\mathcal{S}^{-1}$, we will encounter a term $(\mathcal{S}^{-1})_{12}[(\mathcal{S}^{-1})_{22}]^{-1}(\mathcal{S}^{-1})_{21}$ which is

Figure 2: Diagrams for the gap and pseudogap. The numbers 1 and 2 are the NG indices.

illustrated in Fig. 2(a). We will show that the correction to the fermion self-energy due to fluctuations of di-fermion bosons in Fig. 2(b) contributes to the diagonal parts of the full inverse propagator $\mathcal{S}^{-1}$.

The contribution from Fig. (2b) can be written as

$$\begin{aligned}\Sigma_{11}(\mathcal{D},\mathcal{S}) &\approx \left\{\mathrm{Tr}[D_{ij}\widehat{\Gamma}_i S\widehat{\Gamma}_j]\right\}_{11}\\ &= -16g^2T\sum_n\int\frac{d^3q}{(2\pi)^3}\left[D_{RR}(Q)+iD_{IR}(Q)\right]\gamma_5 S_{22}(P+Q)\gamma_5\\ &= 16g^2T\sum_n\int\frac{d^3q}{(2\pi)^3}\frac{1}{(p_0+\mu_b)^2-E_p^2}\gamma_0\Lambda_{p+q}^{-e}\frac{1}{p_0+q_0-\mu-eE_{p+q}}\end{aligned}\tag{14}$$

where we have taken the bare propagator S and D in (5) as approximations to the full ones. Here $i,j=R,I$ denote the real and imaginary components, q_0 is the bosonic Matsubara frequency $q_0=i2\pi nT$. We also uses the equalities $D_{RR}(Q)=D_{II}(Q)$ and $D_{IR}(Q)=-D_{RI}(Q)$, see App. (A). The energy projector is defined by

$$\Lambda_p^e=\frac{1}{2}\left(1+e\frac{\gamma_0\boldsymbol{\gamma}\cdot\mathbf{p}+\gamma_0 m}{E_p}\right),$$

where $e=\pm$. The presence of $\Sigma_{11/22}$ is equivalent to $(S^{-1})_{11/22}$ gets a correction, where S^{-1} is given by Eq. (5). We can verify it by seeing

$$\begin{aligned}\mathcal{S}_{11} &= \left[(\mathcal{S}^{-1})_{11}-(\mathcal{S}^{-1})_{12}((\mathcal{S}^{-1})_{22})^{-1}(\mathcal{S}^{-1})_{21}\right]^{-1}\\ &= \left[(S^{-1})_{11}-\Sigma_{11}-(S^{-1})_{12}((S^{-1})_{22}-\Sigma_{22})^{-1}(S^{-1})_{21}\right]^{-1}\\ &\approx \left[(S^{-1})_{11}-\Sigma_{11}-(S^{-1})_{12}((S^{-1})_{22})^{-1}(S^{-1})_{21}\right]^{-1}\end{aligned}\tag{15}$$

where Σ_{11} is assumed the value in Eq. (14). Note that in reaching the last line of the above equation we have neglected Σ_{22} for clarity since it is sandwiched by $(S^{-1})_{12}$ and $(S^{-1})_{21}$ and complicates the calculation. We do not include fermion self-energy contribution to Σ_{11}, $\sim\gamma_0 g^2 k_0\ln\frac{M^2}{k_0^2}$ [32, 33] coming from long range gauge interaction such as magnetic photon or gluon exchanges, since the gauge field is absent in the current version of our model. We can easily evaluate

$$\begin{aligned}-\Sigma_{11}-(S^{-1})_{12}((S^{-1})_{22})^{-1}(S^{-1})_{21} &\approx -16g^2T\sum_{n,e}\int\frac{d^3q}{(2\pi)^3}\gamma_0\Lambda_p^{-e}\frac{1}{p_0+q_0-\mu-eE_p}\times\frac{1}{(p_0+\mu_b)^2-E_p^2}\\ &\quad+\sum_e\frac{\Delta^2}{p_0-\mu-eE_p}\gamma_0\Lambda_p^{-e}\approx\sum_e\frac{\Delta^2+\Delta_{pg}^2}{p_0-\mu-eE_p}\gamma_0\Lambda_p^{-e},\end{aligned}\tag{16}$$

where we have assumed that $q\ll p$ so $|\mathbf{p}+\mathbf{q}|\approx p$. The so-called pseudogap Δ_{pg} can be defined [56, 57, 58, 59],

$$\Delta_{pg}^2 = -16g^2T\sum_n\int\frac{d^3q}{(2\pi)^3}\frac{1}{(p_0+\mu_b)^2-(E_p^b)^2}=16g^2\int\frac{d^3q}{(2\pi)^3}\frac{1+f_B(E_q^b-\mu_b)+f_B(E_q^b+\mu_b)}{2E_q^b}$$

$$\sim\ 16g^2 \int \frac{d^3q}{(2\pi)^3} \frac{f_B(E_q^b-\mu_b)+f_B(E_q^b+\mu_b)}{2E_q^b}, \tag{17}$$

where in the last line we remove the divergent part $1/(2E_q^b)$ coming from vacuum. The boson energy is $E_q^b=\sqrt{q^2+m_b^2}$. As lowest order contribution, the role of the DS equation for fermions is approximately equivalent to adding a pseudogap term Δ_{pg}^2 to the condensate square Δ^2 in the dispersion relation of quasi-particles. A few remarks about the pseudogap are in order. Here we define the pseudogap as the correction to fermion selfenergy from Fig. 2(b) at the static limit. Its effect looks like that the real gap square gets a correction, but it is not a real gap. Actually it has analytical structure from which one can compute the density of states from the difermion fluctuation [56, 57].

The renormalized boson mass can be determined by solving the pole equation of the full propagator at static limit with zero gap and pseudogap,

$$\det\mathcal{D}^{-1}(p_0,p)\big|_{p=\Delta=\Delta_{pg}=0} = \det\left[D^{-1}(p_0,p)\big|_{p=0} + \Pi(p_0,p)\big|_{p=\Delta=\Delta_{pg}=0}\right] = 0, \tag{18}$$

where p_0 is a real number with analytic extension. Note that the off-diagonal parts of the boson self-energy are vanishing, $\Pi_{IR}=\Pi_{RI}=0$. The analytical expressions for Π_{RR} and Π_{II} are given in App. (A). When setting $\Delta=0$, we have

$$\Pi_0(p_0) \equiv \Pi_{RR}(p_0,p)\big|_{p=\Delta=0} = \Pi_{II}(p_0,p)\big|_{p=\Delta=0}\,. \tag{19}$$

Eq. (18) is then simplified as

$$\left[(p_0+\mu_b)^2-m_b^2-\Pi_0(p_0)\right]\left[(p_0-\mu_b)^2-m_b^2-\Pi_0(p_0)\right] \;=\; 0. \tag{20}$$

Let us focus on the following equation

$$(p_0+\mu_b)^2-m_b^2-\Pi_0(p_0)=0. \tag{21}$$

In general Eq. (21) can be understood as a complex equation by analytic extension $p_0\to p_0-i\frac{\eta}{2}$ with the width η, then we have

$$\begin{aligned}(p_0+\mu_b)^2-\frac{\eta^2}{4}-m_b^2-\mathrm{Re}\Pi_0(p_0-i\frac{\eta}{2}) \;&=\; 0,\\ -(p_0+\mu_b)\eta-\mathrm{Im}\Pi_0(p_0-i\frac{\eta}{2}) \;&=\; 0.\end{aligned} \tag{22}$$

The renormalized mass for bosons can be defined by

$$m_{br}^2 \equiv (p_0+\mu_b)^2 = m_b^2+\mathrm{Re}\Pi_0(p_0-i\frac{\eta}{2}) \tag{23}$$

with a small positive η. If the boson is stable, then $\eta=0$ and the above becomes

$$m_{br}^2 \equiv (p_0+\mu_b)^2 = m_b^2+\Pi_0(p_0). \tag{24}$$

By setting $\eta=0$, we can determine the dissociation temperature T^* for bosons from the first line of Eq. (22) with $p_0=2m-\mu_b=2(m-\mu)>0$. It is necessary to solve the DS equation in full consistency to get T^*, which is very involved and we reserve it for a future study.

V. GAP AND DENSITY EQUATION

In this section we discuss the gap and density equations which are derived from the thermodynamical potential Ω by taking its derivative with respect to the gap Δ and chemical potential μ.

The gap equation reads,

$$\frac{\partial\Omega}{\partial\Delta} = \left\{\frac{m_b^2-\mu_b^2}{4g^2} - \sum_{e=\pm}\int\frac{d^3k}{(2\pi)^3}\frac{\tanh[\epsilon_k^e/(2T)]}{2\epsilon_k^e} - \frac{\partial\Gamma_{2PI}}{\partial(\Delta^2)}\right\}2\Delta = 0. \tag{25}$$

Here the excitation energy for fermions is given by

$$\epsilon_k^e = \sqrt{(\xi_k^e)^2+\Delta^2}, \tag{26}$$

with the fermion energy $E_k = \sqrt{k^2+m^2}$ and $\xi_k^e = E_k - e\mu$. The 2PI effective potential Γ_{2PI} is analytically evaluated in App. (B). The density equation is obtained by taking derivative of the thermodynamic potential with respect to the fermion chemical potential,

$$\begin{aligned} n &= -\frac{\partial\Omega}{\partial\mu} = \frac{2\mu\Delta^2}{g^2} + 2\sum_{e=\pm}\int\frac{d^3k}{(2\pi)^3}\frac{e\xi_k^e}{2\epsilon_k^e}\left[f_F(\epsilon_k^e) - f_F(-\epsilon_k^e)\right] \\ &+2\sum_{e=\pm}\int\frac{d^3k}{(2\pi)^3} e f_B(E_k^b - e\mu_b) + \frac{\partial\Gamma_{2PI}}{\partial\mu} \end{aligned} \tag{27}$$

We use an effective Fermi momentum p_F to parametrize n with the relation $n = p_F^3/(3\pi^2)$. Throughout the paper we fix the value of the effective momentum $p_F = 0.86$. We can define from the total fermion number density n the density fraction for fermions, condensed and thermal bosons, and the 2PI part as

$$\begin{aligned} \rho_0 &\equiv \frac{2\mu\Delta^2}{ng^2}, \\ \rho_F &\equiv \frac{2}{n}\sum_{e=\pm}\int\frac{d^3k}{(2\pi)^3}\frac{e\xi_k^e}{2\epsilon_k^e}\left[f_F(\epsilon_k^e) - f_F(-\epsilon_k^e)\right], \\ \rho_B &\equiv \frac{2}{n}\sum_{e=\pm}\int\frac{d^3k}{(2\pi)^3} e f_B(E_k^b - e\mu_b), \\ \rho_{\Gamma_2} &\equiv \frac{1}{n}\frac{\partial\Gamma_{2PI}}{\partial\mu}, \end{aligned}$$

which satisfies

$$\rho_0 + \rho_F + \rho_B + \rho_{\Gamma_2} = 1$$

We can treat m_b^2 as a crossover parameter and solve Δ, μ and Δ_{pg} with the gap equation (25), the density equation (27) and the pseudogap definition (17).

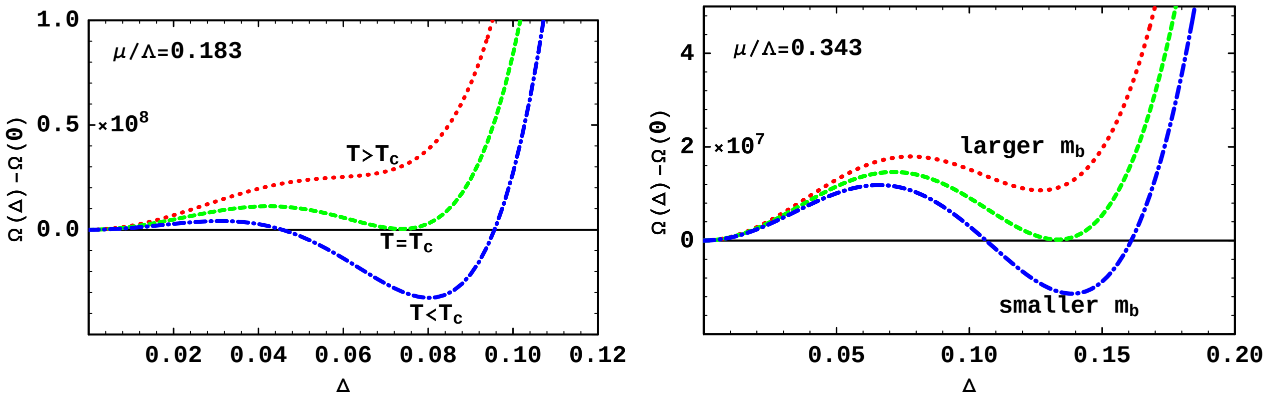
VI. NUMERICAL RESULTS

It is well known that fluctuations may change the superconducting phase transition to be of first-order, such as the intrinsic fluctuating magnetic field in normal superconductors [60], or the gauge field fluctuations in color superconductors [54], due to the fact that fluctuations bring a cubic term of the condensate to the effective potential making the Landau theroy of continuous phase transition invalid. In our case the fluctuations of di-fermions in superfluids also leads to a first-order phase transition. In grand canonical ensemble with chemical potential fixed, as shown in Fig. 3, the thermodynamic potential $\Omega(\Delta)$ as a function of the condensate Δ, has a metastable state if the fluctuation contribution is taken into account. In the left penal with decreasing T, in the right penal with decreasing m_b, we see a sudden jump of the minimum of $\Omega(\Delta)$ from $\Delta = 0$ to a non-vanishing value of Δ, signaling a phase transition from normal phase to a superfluid one. The discontinuity in the order parameter marks the first-order phase transition.

We find that there are three extrema in $\Omega(\Delta)$ near the transition point, which means that there are at least three solutions to the gap equation with fixed chemical potential. So satisfying the gap equation is not sufficient for the ground state which corresponds to the global minimum of the thermodynamic potential.

From the density equation (27), the discontinuity in the gap leads to that of the particle number density. This situation cannot happen in canonical ensemble with fixed particle number. We have checked with the free energy $F = \Omega + \mu n$ as a function of the condensate and found no metastable state. The free energy has only one extremum, which is the global minimum and changes continuously at the transition point. The non-zero solution to the gap equation corresponds to the superfluid phase, the ground state of the system. The Thouless criterion can be employed to determine the transition temperature. Although the order parameter changes smoothly, the phase transition is still a first-order one. We can take the formation of BEC in a free bosonic gas as an example. Below the critical temperature, the chemical potential equals to the ground state energy (zero in non-relativistic or the boson mass in relativistic system), so the entropy per particle $s_1 = -\left.\frac{\partial\mu}{\partial T}\right|_P (T < T_c) = 0$. Above the critical temperature, the chemical potential drops with a negative slope $\left.\frac{\partial\mu}{\partial T}\right|_P (T > T_c) < 0$ giving a non-zero entropy per particle s_2. So

Figure 3: The first order phase transition due to the di-fermion fluctuations.



there is a nonzero latent heat $T_c(s_2 - s_1)$. The same phenomenon occurs to our current case with fixed total particle number.

Now we present the numerical results by solving the gap equation (25), the density equation (27) and the pseudogap equation (17) with fixed total particle number. From the solutions we can also determine the transition temperature T_c at which the condensate Δ turns to zero.

The results for the gap Δ, the fermion chemical potential μ, the pseudogap Δ_{pg} and the density ρ as functions of the crossover parameter m_b are shown in Fig. 4. The parameters are set to $g = 1.8$, $T = 0.14$ and $m = 0.28$. We see that the BEC/BCS regions correspond to the small/large end of m_b. In the BEC region when m_b is small, the particle population is dominated by the BEC part ρ_0 while the fermion part ρ_F is negligible, and there is no Fermi surface characterized by the small fermion chemical potential μ. In the BCS region when m_b is large, the system is mainly fermionic manifested by a clear Fermi surface and large fraction of fermion population. We see that the pseudogap Δ_{pg} is small due to the small temperature chosen. The thermal density fractions ρ_B and ρ_F increases with the growing chemical potential. The contribution from the 2PI part ρ_{Γ_2} is large indicating the strong interaction between fermions and bosons. In the figure, there are breakpoints at $m_b \approx 0.82$ for all curves because the system enters the normal state, i.e. $\Delta = 0$, when m_b is larger. In other words, the breakpoints at $m_b \approx 0.82$ indicates that the transition temperature T_c equals to the temperature parameter $T = 0.14$. When $m_b \lesssim 0.82$, the left area to the break point, we have $T_c \geq T = 0.14$.

In the left panel of Fig. 5, we present the results for T_c, μ (at $T = T_c$) and Δ_{pg} (at $T = T_c$) as functions of m_b; in the right panel we give the results for density fractions ρ_B, ρ_F, and ρ_{Γ_2} all at $T = T_c$ versus m_b. The behavior of T_c is similar to Δ, comparing Fig. 4 and Fig. 5. Although the shape of μ at $T = T_c$ is like μ at $T = 0.14$ in Fig. 4, note that the temperature at each point of the curve of μ at $T = T_c$ versus m_b is different because T_c itself is running with m_b. Comparing to the result for Δ_{pg} in Fig. 4, the result at $T = T_c$ is rather large especially in the BEC region where T_c is large. The population fraction for thermal bosons ρ_B at T_c changes into ρ_{Γ_2} and ρ_F with increasing m_b. ρ_B intersects with ρ_{Γ_2} at $m_b \approx 0.4$ meaning a transition from a bosonic system to a strongly interacting one. It intersects with ρ_F at $m_b \approx 0.9$ meaning a transition to a fermionic interacting system. The intersection of ρ_{Γ_2} and ρ_F occurs at $m_b \approx 1.5$ indicating a transition from a fermionic interacting system to a fermionic system.

The temperature behavior of all quantities in Fig. 4 and 5 are given in Fig. 6 with fixed $m_b = 0.56$ (in BEC regime). It is easy to understand the curve of Δ versus T: above a critical temperature T_c the condensate turns to zero. There is a break point on the curve of μ at T_c, below which μ increases with increasing T but above which it decreases. The pseudogap is a rising function of T. The density fraction ρ_0 is proportional to Δ^2, so its shape is close to that of Δ. As T increases more and more condensed bosons turn to thermal bosons, fermions and interaction part. The thermal boson fraction ρ_B always grows with T. The 2PI part ρ_{Γ_2} is also shown whose effect is large at high T. Due to more sensitive response to increasing temperature for bosons than fermions, the rising feature of ρ_B above T_c can be easily understood. The decreasing of μ_b above T_c is due to particle number conservation.

VII. SUMMARY AND DISCUSSIONS

We investigate the fluctuation effects of di-fermion fields in the BCS-BEC crossover in a relativistic superfluid within the CJT formalism. We found in grand canonical ensemble the fluctuations lead to the normal-superfluid phase transition of first order with metastable states. In canonical ensemble, the phase transition is a first order one

Figure 4: The gap Δ, the fermion chemical potential μ, the pseudogap Δ_{pg} (the left panel) and density fractions for BEC ρ_0, thermal bosons ρ_B, fermions ρ_F, and 2PI part ρ_{Γ_2} (the right panel) as functions of m_b with the fixed coupling constant $g = 1.8$, the temperature $T = 0.14$ and the fermion mass $m = 0.28$.

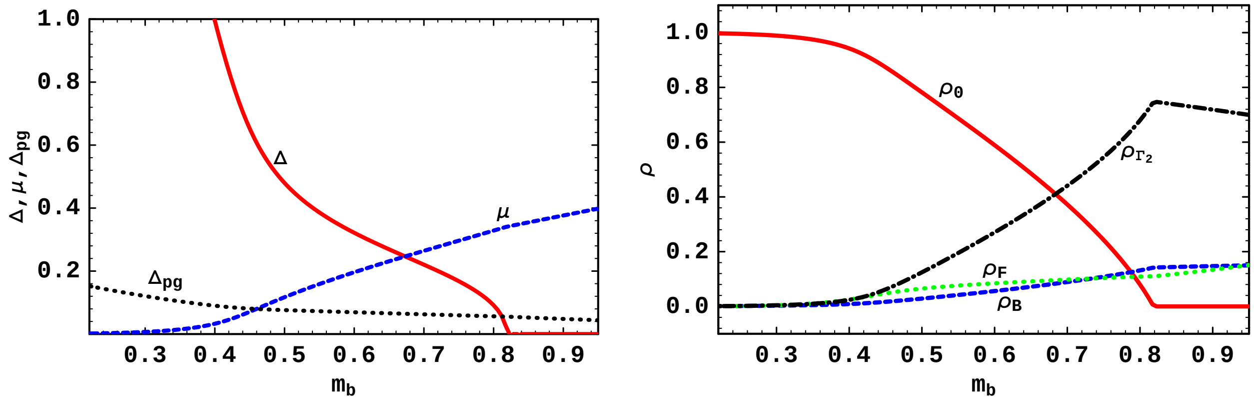
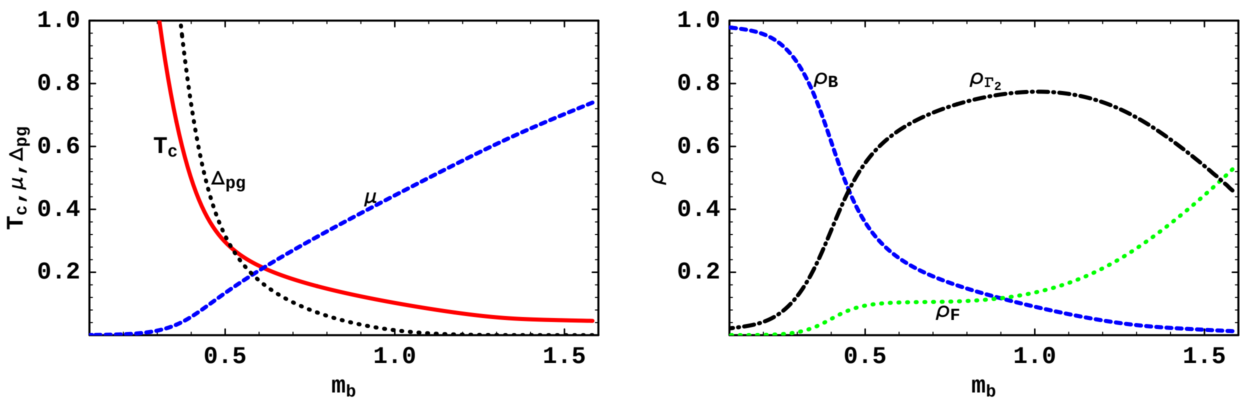


Figure 5: Left panel: the transition temperature T_c, the fermion chemical potential μ at $T = T_c$, and the pseudogap Δ_{pg} at $T = T_c$ versus m_b. Right panel: density fractions for thermal bosons ρ_B, fermions ρ_F, and the 2PI part ρ_{Γ_2} all at $T = T_c$ versus m_b. The parameters are $g = 1.8$ and $m = 0.28$.



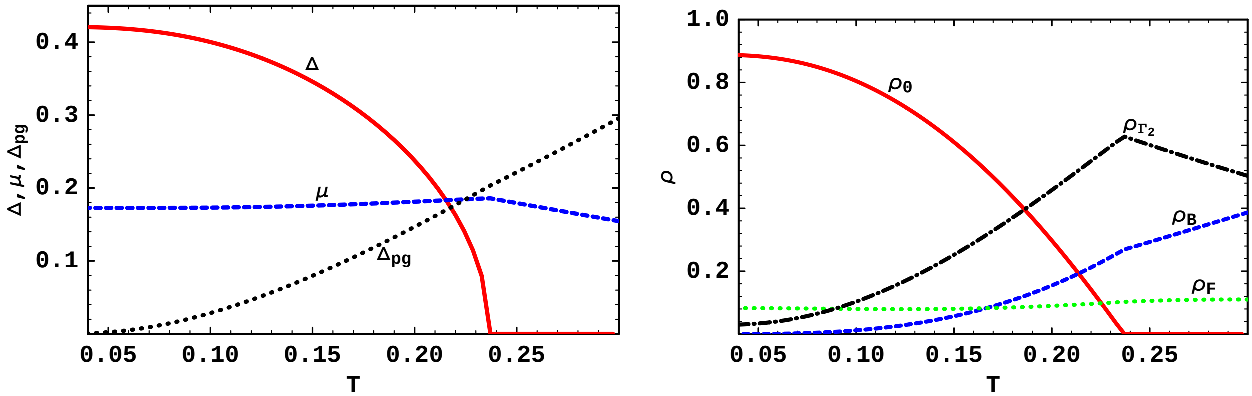
too but without metastable states. The order parameter changes smoothly near the transition point which ensures the validity of the gap equation and the Thouless criterion.

In this work, we use the bare boson mass squared m_b as the crossover parameter. We find that the system is in the BCS/BEC regime for large/small m_b. Comparing to the results in our previous work [49] with the crossover parameter $x = -\frac{m_{br}^2 - \mu_b^2}{4g^2}$ where negative/positive x defines the BCS/BEC regime, we find that these two parameters are equivalent in describing the crossover behavior. We can verify numerically that there is a one-to-one mapping between m_b and x. In this paper, we choose m_b as it is more simple. The gap equation is derived with fixed m_b instead of x which is regarded as a derivative parameter.

We see that the pseudo-gap Δ_{pg} and the 2PI contribution to particle density ρ_{Γ_2} vanish as the temperature goes to zero. Therefore the results at low temperatures reproduce those in the mean field approximation. This is because as the temperature goes to zero the interaction between thermal bosons and fermions is switched off which results in vanishing fluctuation contributions. At finite temperature and in the middle of the crossover, ρ_{Γ_2} is dominant indicating the strong coupling between bosons and fermions. In the BEC or BCS end the number density is dominated by bosons or fermions respectively, during the crossover from BCS to BEC, the system undergoes a strongly interacting intermediate stage. This picture is a little different from that in the mean field approach where bosons and fermions are transformed to each other directly.

Since the current model is a relativistic one, the anti-particles should appear in some circumstances. In the left panel of Fig. 4, we see that $\mu = T = 0.14$ at $m_b \approx 0.5$, which means the thermal energy is comparable to the energy needed for exciting a pair of fermion and anti-fermion. At the deep end of BEC region the fraction of anti-particles is not negligible, which is partially responsible for the large condensate. In the left panel of Fig. 5, at $m_b \approx 0.6$ where $T_c = \mu$, the appearance of anti-particles drives the critical temperature and the pseudo-gap to be large. As expected the fluctuation effects grow with increasing temperatures. The pseudo-gap is a monotonous increasing function of the

Figure 6: Left panel: the condensate Δ, the fermion chemical potential μ, and the pseudogap Δ_{pg} versus T. Right panel: density fractions for condensed and thermal bosons ρ_0 and ρ_B, fermions ρ_F, and the 2PI part ρ_{Γ_2} versus T. The parameters are set to $g = 1.8$ and $m = 0.28$ and $m_b = 0.56$.



temperature, whose magnitude is seeable at critical temperature in the BEC regime.

In this work, we expressed the effective potential in terms of bare propagators. The role of the DS equations are taken by the pseudo-gap and the renormalized boson mass. There is another approach of using the full propagators to construct the effective potential, where the complete form of the renormalization can be implemented. Another hard but interesting problem to solve the DS equations with full self-consistency. All these problems deserve a detailed study in the future.

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Appendix A: BOSON PROPAGATOR AND SELF-ENERGY

The bare propagator for bosons can be found from Eq. (5),

$$D = -\frac{1}{\det(D^{-1})}\begin{pmatrix} P_\mu P^\mu + \mu_b^2 - m_b^2 & -2\mu_b i p_0 \\ 2\mu_b i p_0 & P_\mu P^\mu + \mu_b^2 - m_b^2 \end{pmatrix}, \tag{A1}$$

where

$$\det(D^{-1}) = [(p_0+\mu_b)^2 - (E_p^b)^2][(p_0-\mu_b)^2 - (E_p^b)^2] = [p_0^2 - (\mu - E_p^b)^2][p_0^2 - (\mu + E_p^b)^2]. \tag{A2}$$

We see that $D_{RR}(Q) = D_{II}(Q)$ and $D_{IR}(Q) = -D_{RI}(Q)$.

The boson self-energy matrix

$$\Pi_{ij}(\mathcal{S},\mathcal{D}) \approx \frac{1}{2}\mathrm{Tr}[\widehat{\Gamma}_i S \widehat{\Gamma}_j S] \tag{A3}$$

are evaluated as

$$\begin{aligned} \Pi_{RR} &\approx \frac{1}{2}\mathrm{Tr}[\widehat{\Gamma}_R S \widehat{\Gamma}_R S] = -8g^2\left[\mathrm{Tr}(\gamma_5 S_{11}\gamma_5 S_{22}) + \mathrm{Tr}(\gamma_5 S_{21}\gamma_5 S_{21})\right], \\ \Pi_{II} &\approx \frac{1}{2}\mathrm{Tr}[\widehat{\Gamma}_I S \widehat{\Gamma}_I S] = -8g^2\left[\mathrm{Tr}(\gamma_5 S_{11}\gamma_5 S_{22}) - \mathrm{Tr}(\gamma_5 S_{21}\gamma_5 S_{21})\right], \\ \Pi_{RI} &= \Pi_{IR} = 0, \end{aligned} \tag{A4}$$

where

$$\begin{aligned}
\mathrm{Tr}[\gamma_5 S_{11}\gamma_5 S_{22}] &= -T\sum_n\int_q \frac{p_0-\mu+eE_p}{p_0^2-(\mu-eE_p)^2-\Delta^2}\times\frac{p_0+q_0+\mu+e'E_{p+q}}{(p_0+q_0)^2-(\mu+e'E_{p+q})^2-\Delta^2}\mathrm{Tr}[\Lambda_p^e\Lambda_{p+q}^{-e'}],\\
\mathrm{Tr}[\gamma_5 S_{21}\gamma_5 S_{21}] &= \mathrm{Tr}[\gamma_5 S_{12}\gamma_5 S_{12}]\\
&= -T\sum_n\int_q \frac{1}{q_0^2-(\mu-eE_q)^2-\Delta^2}\times\frac{1}{(p_0+q_0)^2-(\mu-e'E_{p+q})^2-\Delta^2}\Delta^2\mathrm{Tr}[\Lambda_q^{-e}\Lambda_{p+q}^{-e'}]. \quad \text{(A5)}
\end{aligned}$$

The results for $\Pi(p_0,p)$ are

$$\begin{aligned}
\Pi_{RR}(p_0,p) &= \Pi_0+\Pi_1\\
\Pi_{II}(p_0,p) &= \Pi_0-\Pi_1
\end{aligned}$$

where

$$\begin{aligned}
\Pi_0(p_0,p) &= -8g^2\sum_{e,e',e_1,e_1'}\int\frac{d^3q}{(2\pi)^3}\mathrm{Tr}[\Lambda_q^e\Lambda_{p+q}^{e'}]\frac{\exp\left[\beta(-p_0+e_1'\epsilon_{p+q}^{e'}-e_1\epsilon_q^e)\right]-1}{-p_0+e_1'\epsilon_{p+q}^{e'}-e_1\epsilon_q^e}\\
&\times f_F(-e_1\epsilon_q^e)f_F(e_1'\epsilon_{p+q}^{e'})\frac{(\epsilon_{p+q}^{e'}-e_1'\mu+e'e_1'E_{p+q})(\epsilon_q^e+e_1\mu-ee_1E_q)}{4\epsilon_{p+q}^{e'}\epsilon_q^e}\\
\Pi_1(p_0,p) &= -8g^2\Delta^2\sum_{e,e',e_1,e_1'}e_1e_1'\int\frac{d^3q}{(2\pi)^3}\mathrm{Tr}[\Lambda_q^e\Lambda_{p+q}^{e'}]\frac{\exp\left[\beta(-p_0+e_1'\epsilon_{p+q}^{e'}-e_1\epsilon_q^e)\right]-1}{-p_0+e_1'\epsilon_{p+q}^{e'}-e_1\epsilon_q^e}\\
&\times f_F(-e_1\epsilon_q^e)f_F(e_1'\epsilon_{p+q}^{e'})\frac{1}{4\epsilon_{p+q}^{e'}\epsilon_q^e} \quad \text{(A6)}
\end{aligned}$$

where the quasi-particle energy for fermions is given by Eq. (26). Note that in renormalized quantities/constants and the effective potential Γ_{2PI} the full fermion propagators are used, which amounts to taking the pseudogap contribution into account, thus we have

$$\epsilon_k^e\to\sqrt{(\xi_k^e)^2+\Delta^2+\Delta_{pg}^2}. \quad \text{(A7)}$$

In this and the next appendix, we always imply the above replacement.

Appendix B: 2PI EFFECTIVE POTENTIAL

In this appendix we give the 2PI effective potential Γ_{2PI} using the boson self-energy in Eq. (A4) as follows,

$$\begin{aligned}
\Gamma_{2PI} &= -\frac{1}{2}\mathrm{Tr}(D_{RR}\Pi_{RR})-\frac{1}{2}\mathrm{Tr}(D_{II}\Pi_{II})=-\mathrm{Tr}(D_{RR}\Pi_0)\\
&= -T\sum_n\int\frac{d^3p}{(2\pi)^3}\int_0^\beta d\tau' D_{RR}(\tau',p)e^{p_0\tau'}\int_0^\beta d\tau\Pi_0(\tau,p)e^{p_0\tau}\\
&= -\int\frac{d^3p}{(2\pi)^3}\int_0^\beta d\tau D_{RR}(\beta-\tau,p)\Pi_0(\tau,p)\\
&= 4g^2\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\sum_{e,e',e_1,e_1',e_2,e_3}\mathrm{Tr}[\Lambda_q^e\Lambda_{p+q}^{e'}]\frac{\exp[\beta(e_3\mu_b+e_2E_p^b+e_1'\epsilon_{p+q}^{e'}-e_1\epsilon_q^e)]-1}{e_3\mu_b+e_2E_p^b+e_1'\epsilon_{p+q}^{e'}-e_1\epsilon_q^e}\\
&\times f_B(e_3\mu_b+e_2E_p^b)f_F(-e_1\epsilon_q^e)f_F(e_1'\epsilon_{p+q}^{e'})\frac{e_2}{2E_p^b}\cdot\frac{\epsilon_{p+q}^{e'}-e_1'\mu+e'e_1'E_{p+q}}{2\epsilon_{p+q}^{e'}}\cdot\frac{\epsilon_q^e+e_1\mu-ee_1E_q}{2\epsilon_q^e}, \quad \text{(B1)}
\end{aligned}$$

where we have used

$$\Pi_0(\tau,p) = T\sum_n\Pi_0(p_0,p)e^{-p_0\tau}$$

$$= -8g^2 \sum_{e,e',e_1,e_1'} \int \frac{d^3q}{(2\pi)^3} \mathrm{Tr}[\Lambda_q^e \Lambda_{p+q}^{e'}] e^{(e_1' \epsilon_{p+q}^{e'} - e_1 \epsilon_q^e)(\beta-\tau)}$$

$$\times f_F(-e_1\epsilon_q^e) f_F(e_1'\epsilon_{p+q}^{e'}) \frac{\epsilon_{p+q}^{e'} - e_1'\mu + e'e_1' E_{p+q}}{2\epsilon_{p+q}^{e'}} \cdot \frac{\epsilon_q^e + e_1\mu - ee_1 E_q}{2\epsilon_q^e}, \tag{B2}$$

and

$$D_{RR}(\tau,p) \;=\; \sum_{e_2,e_3} \frac{e_2}{4E_p^b} f_B(e_3\mu_b + e_2 E_p^b) e^{(e_3\mu_b + e_2 E_p^b)\tau}. \tag{B3}$$

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